Aerodynamic Characteristics of Slender Wings with Sharp Leading Edges—A Review

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This paper presents an overview of the current state-of-the-art regarding slender wings with sharp leading edges; i.e., wings characterized by the presence of leading edge separation at most angles of attack. Several theoretical methods are discussed in detail and their results are compared with experimental data. Both steady and some unsteady flows are considered. No one theory adequately predicts all aspects of the flow process, and more work is needed, particularly in the fields of vortex control and unsteady flow.

Nomenclature

$A\!\!R$	= aspect ratio
c	= centerline chord
C_N	= normal force coefficient
C_P	= pressure coefficient
s	= semispan of wing
t	= time
U	= freestream velocity
x,y,z	= rectangular cartesian coordinates
(X_{cp}/c)	= nondimensional location of the center of pressure
α	= angle of attack
ω	= circular frequency, rad/sec

I. Introduction

THE flow about slender wings with sharp leading edges in steady motion is well-known.[1-3] Upper and lower surface boundary layers separate from the wing along each leading edge to form free shear layers, and then proceed to roll up into spiral vortices lying above the wing and inboard of the leading edges (Fig. 1).

At low angles of attack, there is a reattachment line OB on the upper surface. Inboard of OB the streamlines are mainly in the streamwise direction. Outboard of OB the flow beneath the separated vortex travels towards the leading edge. With increasing angle of attack the attachment line OB moves inboard.

The separated vortex induces adverse pressure gradients of sufficient strength to cause the boundary layer outboard of the vortex to separate again (OC) forming a secondary vortex of opposite circulation to the primary vortex.

Outboard of the secondary line OC, the pressure gradients and surface shears are small and it is difficult to visualize the surface flow pattern by standard techniques. However, a slight inboard trend under the secondary vortex has been observed.[1,2,4]

The possibility of a tertiary separation has been discussed by Lawford[2] and Hummel,[5] but whether or not it occurs is difficult to verify experimentally, particularly at high Reynolds numbers when the boundary layer ahead of secondary separation is turbulent.

At high angles of attack the phenomenon of vortex breakdown, i.e., the bursting of the tightly rolled vortex core, is observed above the wing surface.[6-8] Associated with the rapid increase in size of the vortex core are a loss of lift, pressure fluctuations, and a general unsteadiness. In unsteady flow, both the position and strength of the primary vortex vary (for example Refs. 9 and 10) because of changes in the shedding rate of leading edge vorticity.

The volume of work, both theoretical and experimental, that has been undertaken in the field of slender wings with sharp leading edges is considerable, and a complete review of all relevant data would be impossible in a paper of this length. Therefore, only a representative sample of the available material is referenced and discussed. For further details the reader should consult the more recent references listed, which cross-reference work that is not mentioned here.

II. Theoretical Methods for Steady Flow

A. Linearized Lifting Surface Theory

Classical linearized lifting surface theory is applicable to thin wings with small camber, twist, and angle of attack with attached flow. The solution for the lifting problem due to camber and angle of attack can be obtained by replacing the wing, and its trailing wake, by a plane vortex sheet of bound trailing components. The induced upwash at any point on the wing can be determined, and by applying the boundary condition of tangential flow, an integral equation can be obtained, which can be reduced (e.g., Bisplinghoff et al.[11]) to a form for developing approximate solutions for wings of low aspect ratio.

B. Linear Slender Wing Theory

Jones[12] simplified the fundamental equations for wings of small aspect ratio to obtain an analytic expression for the wing loading. He applied conformal transformation techniques to each crossflow plane which were assumed to act independently of each other.

Identical results can be obtained by applying slenderness assumptions to the basic equations and by using them to obtain the wing loading, which then is integrated to give the normal force coefficent C_N. This is plotted in Fig. 2 for a unit aspect ratio wing. Trailing edge corrections have applied[13] to the Jones theory, reducing the predicted lift and bringing it more into line with the numerical results for lifting surface theories.

C. Complete Numerical Solutions

Several numerical techniques are available for solving the basic integral equation,[14-16] but only the Multhopp[14] solution is considered here. The equation is first modified so that the double integral is over the wing only; it then is replaced by a matrix equation containing values of lift and upwash at a number of specified points on the wing. A minimum number of two chordwise stations must be used in order to obtain both

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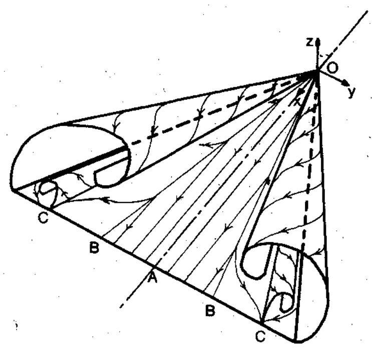

Fig. 1 General flow pattern.

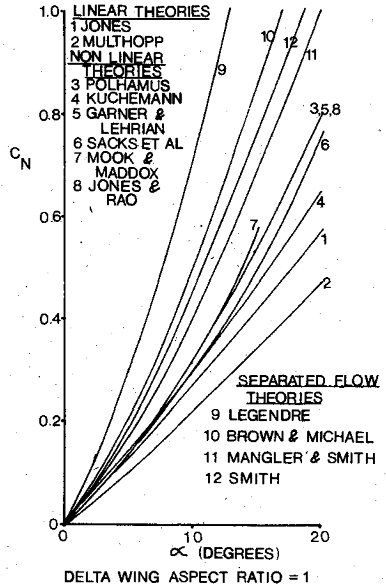

Fig. 2 Theoretical normal force coefficients.

lift and moment. Initial attempts to use a greater number led to integration errors, which only recently have been overcome.[17]

When the aforementioned method is applied to swept wings, uncertainties arise because of the kink in planform. The interpolation formulae that are used give rise to logarithmic infinities, and the planform is normally rounded off to overcome these; errors that are introduced are minimized by requiring the area of the interpolated wing to equal the geometrical area. Figure 2 shows the overall normal force for a unit aspect ratio delta wing obtained by Multhopp's method.

D. Nonlinear Theories

As described, the flow past slender wings with sharp leading edges separates at the leading edges for all normal angles of attack. The concentrated vorticity above the wing induces high suction pressures on the wing's upper surface, resulting in a nonlinear contribution to the wing loading. Generally speaking, theories that allow for the nonlinearities fall into the following three categories.

1) Leading Edge Suction Analogy

Polhamus[18] assumed that, for slender wings where the flow separates at the leading edges and reattaches on the wing's upper surface, the overall lift force can be estimated from the sum of potential and vortex lift contributions. The potential lift contribution is defined as the lift due to potential flow about the wing with zero leading-edge suction. Since reattachment occurs, it is argued that this is the only loss in the potential flow lift.

Applying both the Kutta-Joukowski theorem and the tangential flow conditions, the normal force can be determined.

Vortex lift is caused by the large suction forces induced by the vortices, and is assumed to be equivalent to the leading-edge suction force in potential flow, except that it acts normal to the upper surface. The total C_N on the wing is given by the addition of the potential and vortex components, which can be determined from any lifting surface theory.

Although the analogy is not rigorous, C_N obtained by this method for delta wings shows good agreement with experimental values (Fig. 2). More recently, Polhamus[19] has extended his theory to include double delta and arrow wing planforms at both subsonic and supersonic speeds.

2) Nonlinear Surface Theories

Kuchemann[20] developed a nonlinear slender wing conical flow method by assuming a model consisting of a vortex sheet that incorporates part-span vortices, whose height above the wing is unimportant. The spanwise position of the vortices is given empirically, and it is assumed that the flow separates smoothly from the leading edges so that the vorticity at that point is zero. The tangential flow condition is satisfied for the mean spanwise downwash.

Two infinities at the positions of the part-span vortices occur in the spanwise load distribution. In spite of these two singularities, the load distribution is closer to the experimental distribution than to the linearized methods. C_N derived for a delta wing is shown in Fig. 2.

Bollay[21] suggested a method for low aspect ratio rectangular wings in which all of the vorticity is shed from the wing tips, not from the trailing edge, and is assumed to be along straight lines at an angle to the wing. A solution is obtained by satisying the condition of tangential flow along the wing centerline, and requiring the trailing vorticity that is shed from the wing tips to lie along local streamlines.

Gerston[22] and Garner and Lehrian[23] extended Bollay's model to cover wings of arbitrary shape. Vorticity is assumed to be shed from all points on the wing at an angle $\alpha/2$ to the wing, i.e., the shed vorticity does not lie along streamlines. A rotational wake of finite cross section results. If the wing is regarded as a series of lifting elements, Bollay's theory can be applied. A Multhopp-type lifting surface method was used to solve the integral equations.

There is little physical justification for the model, since the shed vorticity is unlikely to bear much resemblance to the physical rolled-up situation. The spanwise load distribution, therefore, cannot be expected to be reasonable. Curves of C_N and the position of the center of pressure (X_{cp}/c) are shown in Figs. 2 and 3.

Another extension of Bollay's idea has been presented by Sacks, Neilson, and Goodwin.[24] The model assumes that a wing of arbitrary planform can be considered as a series of high-aspect-ratio elements, each having a system of trailing vorticity as well as separation from the tips. The separated sheet from the tips is assumed to arise from a distribution of horseshoe vortices, and makes an angle (chosen to give good agreement with experimental results) with the wing. An additional lifting line is assumed to be situated on the 1/4-chord line of each element, together with its associated trailing vorticity in the plane of the element. Conditions of finite velocity around the tips of each element and tangential flow on the 3/4-chord line of each element are satisfied.

Sacks et al. suggested that, for a unit ratio wing, a value of shedding angle $= 0.8\ \alpha$ should be used. The C_N vs α curve for a delta wing has been obtained[25] by using this value, and is shown in Fig. 2.

Mook and Maddox[26] have used an extension of the vortex lattice method to account for leading-edge separation. In addition to standard vortex lattice, a series of kinked vortex lines (approximately in the local stream direction) are assumed to be shed from the leading edge. An iterative procedure is used to determine the strength and local flow directions with the appropriate boundary conditions. The C_N vs α for a unit aspect delta is plotted in Fig. 2.

Jones and Rao[27] represent the wing with a doublet distribution and allow for flow separation with a series of vortex lines shed from the leading edge. These vortex lines are inclined to the surface at an angle which is a function of angle of attack. Their strength is porportional to the doublet strength at the particular leading-edge box and the constant of proportionality, a function of aspect ratio and angle of attack, is chosen to give good agreement with experimental results. Plots of C_N vs α for a unit aspect ratio delta wing are shown in Fig. 2.

3) *Detached Flow Methods*

Although some of the nonlinear methods described provide reasonable approximations to C_N, only one[26] of them allows for the rolling-up process of the vortex sheets. Thus, the predicted spanwise load distribution generally are unsatisfactory.

Legendre[28] assumed a model with two concentrated vortices lying above the wing and inboard of the leading edges. Two boundary conditions were imposed; velocities along the leading edges must be finite and the total force on each isolated vortex must be zero. On a closed path around the vortex, the velocity potential will have multiple values, which change with the circulation for each complete circuit of the vortex; a cut, or cuts, therefore, must be introduced into the flow. Initially Legendre assumed a cut joining the two vortices, but Adams[29] suggested two cuts joining the concentrated vortices to the adjacent leading edges. This is the model used by Brown and Michael.[30] Since conical flow is assumed, the strength of the vortices increases with x, the increase being fed via the "cuts." The boundary conditions are: tangency of flow on the wing surface, finite velocity at the leading edges, and zero resultant force on the vortex plus cut combination.

The spanwise pressure distribution obtained by the method is shown in Fig. 4 for a unit aspect ratio delta wing at 10° angle of attack. The total force on the wing was obtained from momentum considerations, and is plotted in Fig. 2, together with Legendre's results. The effect of angle of attack on the vortex locations is shown in Fig. 5.

Two physically undesirable features are inherent in the method; the pressure is not continuous throughout the fluid and there is a finite loading at the leading edge. In an attempt to improve the model, Mangler and Smith[31] assumed that the vortex sheets are shed tangentially at the leading edges and roll up as spiral vortices. Each spiral vortex is considered in two distinct parts; an inner part, small in size and remote from the wing, and an outer part, comprising a finite vortex sheet, joining the inner part to the leading edge. The inner part is replaced by a concentrated vortex joined to the outer spiral by a cut. The boundary condition of zero pressure discontinuity across the separated vorticity is modified in the inner region to one of zero total load on the line vortex and cut.

A conformal transformation technique is applied again, but because of the mathematical complexity, the boundary conditions on the outer vortex sheet are satisfied at only one or two discrete points. The variation of C_N and vortex position are shown in Figs. 2 and 5. In order to check whether discrepancies between results from the theory and experiment are caused by the mathematical approximations or by fundamental inadequacies in the model, Smith[32] relaxed the restrictions on the outer part of the vortex sheet and used iterative procedures to obtain a solution. Variation of C_N, spanwise loading, and vortex positions for a delta wing are shown in Figs. 2, 4, and 5.

E. Methods for Nonconical Flows with Separation

Except for delta wings at supersonic speeds, conical flow never occurs, because of nonconical planform shapes, trailing edge effects, and camber effects. In the case of wings with

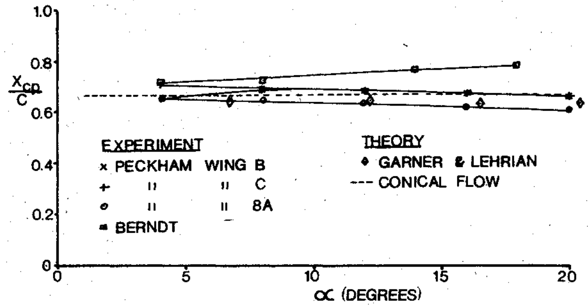

Fig. 3 Position of center of pressure.

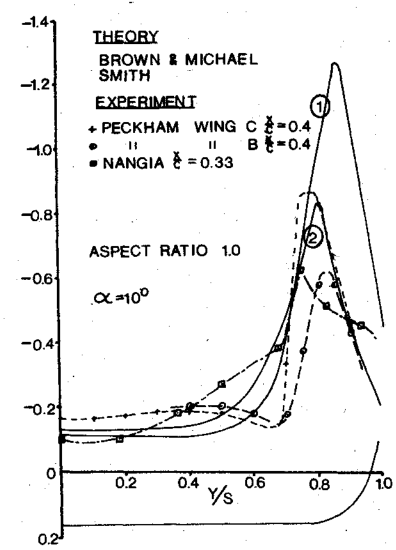

Fig. 4 Spanwise pressure distributions.

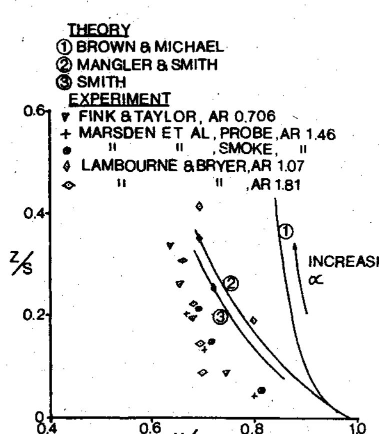

Fig. 5 Positions of vortex cores.

curved leading edges, Smith[33] has extended the Brown and Michael method. Although the overall flow is no longer conical, the solution is based on the assumption that the flow in the vicinity of the apex is conical. The strength and position of the separated vortex near the apex are taken from the Brown and Michael solution. The rate of change of vortex position with x is obtained and used to determine the position and strength a small distance downstream. A step-by-step procedure establishes the flow pattern up to the trailing edge.

In an attempt to formulate a method for an arbitrary wing at subsonic speeds, Nangia and Hancock[34] combined conventional lifting surface theory with the detached flow of Brown and Michael.

In the model, two concentrated vortices lie above the wing, connected to the leading edges by feeding cuts. The trailing wake consists of the vorticity shed from the trailing edge and the continuing concentrated vortices. The trailing edge vorticity is deflected outwards by the velocity field of the concentrated vortices, and at some distance downstream, this vorticity reaches the side edges of the wake, where it is assumed to be convected instantly into the main vortex.

In order to determine the separated vortex strengths, positions, and the wing vorticity distributions, the conditions of tangential flow and zero trailing edge load are applied at a finite number of collocation points, together with the conditions of zero loading on the vortex and cut. Although the numerical work is large, and all of the inherent faults of the Brown and Michael model are present, the estimated chordwise load distribution is in good quantitative agreement with, and the spanwise load distribution is at least representative of experimental results.

III. Experimental Results in Steady Flow

A vast amount of experimental work has been undertaken to obtain details of the changes in flow patterns and load characteristics with variation of the main parameters (e.g., aspect-ratio, planform and thickness distributions). However, direct comparisons between the results of different workers is difficult, since few workers have tested identical models.

A. Balance Measurements

Low-speed balance measurements have been made by many workers, but only the results of Peckham,[1] Earnshaw and Lawford,[2] Fink and Taylor[35] and Berndt[36] will be discussed here, since they are fairly representative, and most of them did work on a unit aspect ratio delta wing, which is used as a comparison basis here.

Peckham carried out tests to determine the effects of aspect ratio and thickness distribution on a series of delta and gothic wings, all with sharp leading edges. Detailed pressure plotting, balance and surface flow visualization results were presented. Reynolds numbers covered the range 2.3×10^6 to 8.6×10^6, but no significant variation with Reynolds number was observed. Variation of position of the center of pressure and C_N with α for three unit aspect ratio delta wings are shown in Figs. 3 and 6.

Earnshaw and Lawford also used wings having plano-convex chordwise sections and aspect ratios 1 and above. Three component balance results, surface flow visualizations and positions of vortex breakdown were presented. The variation of C_N with α is shown in Fig. 6.

Fink and Taylor tested a flat plate delta of aspect ratio 0.706, thickness 0.5%, having an 8.5° chamfer at the leading edges. The slenderness of the wing was chosen to minimize trailing edge effects. Detailed pressure plotting results and total head traverses in the vicinity of the vortex were presented, as well as three component balance measurements.

Some of the effects of round leading edges are shown in Berndt's results. Berndt made three component balance measurements on a series of 10% thick delta and cropped delta wings of aspect ratio 0.5 to 2.5 at Reynolds number of 2×10^6. Variations of position of the center of pressure and C_N with α for a unit aspect ratio wing are shown in Figs. 3 and 6.

All the $C_N \sim \alpha$ curves have the expected nonlinear trends associated with detached flow. Measurements for the sharp edged wings show a trend for the value of C_N to be reduced with increasing thickness. Berndt's results lie below those of other workers, and indicate that primary separation is delayed by the round leading edges.

Figure 3 shows the positions of the center of pressure obtained from balance measurements and theory for a unit aspect ratio wing. A trend is shown for the center of pressure to move aft with increasing thickness for sharp-edged wings, whereas it lies still further aft for round edged wings. No great variation with angle of attack for the sharp-edged wings is shown, but it moves aft with increasing α for the round nosed wing.

B. Pressure Distributions

Peckham, Fink, and Taylor and Nangia[37] all obtained detailed pressure distributions on sharp-edged models. Typical spanwise pressure variations are shown in Fig. 4, together with theoretical distributions. From his results, Peckham determined that at low angles of attack the effect of thickness was to localize the leading-edge separation, leaving the flow external to the boundary layer relatively undisturbed. This implies that the coiled vortex sheets do not develop as rapidly for thick wings as they do for flat plate wings. Peckham also concluded that increasing aspect ratio, thickness, and convexity of the leading edge all have the effect of moving the vortex system outboard.

Nangia integrated his pressure results to obtain C_N values both overall and in the apex retion, where the flow could be assumed to be conical (Fig. 6). As expected, the local C_N values in the apex region lie well above the balance measurements, whereas the overall C_N values are within 5% of Peckham's wing 8A results.

C. Vortex Positions

Measurements of the position of the primary vortex have been made by Fink and Taylor, Lambourne and Bryer[38] and Marsden, Simpson, and Rainbird.[39] Positions were found mostly by total head tube traverses. However, Marsden et al. checked their results by flow visualization using smoke filaments, and Lambourne and Bryer undertook visualization tests in a water tunnel.

Variations of vortex position are shown in Fig. 7, together with theoretical predictions. Agreement is good between the results of Fink and Taylor, Marsden et al. and the high aspect ratio wing of Lambourne and Bryer. Results for the low aspect ratio wing of Lambourne and Bryer lie above and further outboard of the other results, probably because of tunnel constraint effects.

IV. Comparison of Theoretical and Experimental Results

The linear theories of Multhopp and Jones predict values of C_N that are too low, but agreement with Berndt's results for round nosed models is good up to $\alpha = 7°$. Spanwise load distributions are unrealistic with infinities of loading occurring along the leading edges. However, estimation of the chordwise loading predicted by Multhopp's solution is reasonable.

Reasonable estimates of C_N, when compared with experimental results for thin wings (e.g., Nangia[37] of Peckham's wing 8A), are obtained by the nonlinear lifting surface theory of Garner and Lehrian. Hence, there is at least an excuse, if not a rigorous justification, of the model. The position of the center of pressure on a unit aspect ratio delta wing predicted by the method of Garner and Lehrian is in good agreement with measured values of Peckham's wing 8A, and as expected is slightly ahead of the position predicted by conical flow theories.

Good agreement with experimental results for thin wings is also shown by the theories of Sacks et al. and Jones and Rao, but is not surprizing, since the shedding angle of the leading vorticity in one and strength of shed vorticity in the other were chosen as that necessary to give good agreement. One improvement of the Sacks et al. and Jones and Rao models over the Garner and Lehrian model is that vorticity is not shed from all points on the wing surface.

A good estimation of C_N and the chordwise load distributions[40] by the method of Polhamus supports the validity of his leading edge suction analogy. However, extension of the method to wings with curved leading edges is difficult because of inaccuracies in predicting local values of the leading-edge suction force.

Most of the nonlinear conical flow methods overestimate the total loads, and only can be used for comparison in the apex region, where trailing edge effects are small. In this region, agreement for C_N between Nangia's[37] integrated pressures and the method of Mangler and Smith is excellent. Spanwise distribution by the method of Brown and Michael and that of Smith are shown in Fig. 4, and are compared with experimental distributions obtained by Peckham and Nangia. A large, narrow, suction peak that occurs directly under the concentrated vortex characterizes the Brown and Michael solution, whereas the outer vortex sheet of Smith's model has the effect of broadening and lowering the peak. Agreement between the theoretical loading predicted by Smith and experimental distributions is good; remaining differences probably are caused by the effects of secondary separation.

A comparison of the vortex positions obtained by theory and experiment is shown in Fig. 5. The theories of Brown and Michael, and Mangler, and Smith both tend to position the vortices further outboard than the measured values. As might be expected from its closer representation of the flow, Smith's theoretical values lie closest to the experimentally observed vortex positions.

V. Vortex Bursting

At low angles of attack, the leading edge vortices produced by slender wings travel some distance downstream before bursting. An increase in the angle of attack or aspect ratio moves this burst point closer to the trailing edge[6-8] until the burst occurs above the wing, causing a decrease in lift, increase in pitching moment, and buffeting.

If the onset of vortex bursting could be delayed distinct improvements in landing performance would be possible. Some of the work done in this area is discussed in the following.

Bradley and Wray[41] used spanwise blowing and tested three sharp-edged models, including a delta wing. Results for the delta wing (aspect ratio 2.3) without blowing indicate that the vortex burst reaches the trailing edge at approximately 14.5° angle of attack of attack, and the lift falls increasingly below Polhamus' predictions for angles greater than 10°. However, with spanwise blowing, the full Polhamus predictions could be realized up to 27°. How much of this increased lift is due to delaying vortex bursting and how much to the injection of high speed, i.e., low pressure, air over the wing has yet to be resolved.

Dixon, Theisen, and Scruggs,[42] working with higher aspect ratio swept wings, came to a similar conclusion but also suggested that more than one nozzle might be needed. In addition to their considerable experimental work, Dixon et al. attempted a theoretical solution, including blowing, by a vortex lattice method. Because of the complexity of the problem and the simplicity of the model (even the feeding cuts were eliminated to make the model tractable), the method was necessarily very empirical.

Rather than blowing, Hummel[5] found that on a sharp-edged delta wing of $Æ = 1$ the angle of attack at which vortex bursting moved onto the wing could be delayed considerably and the normal force increased by applying suction to the vortex cores downstream of the wing's trailing edge. Suction was applied with two tubes, which could be moved laterally and vertically to locate the vortex core. Although the results were encouraging, Hummel found that even with suction he could not really improve on the results where no suction device at all was present, indicating large interference due to the presence of the suction tubes themselves.

Another approach to vortex control has been used on the Saab Viggen. A small low aspect ratio canard is mounted ahead of the main wing and generates a stable vortex (because of its low aspect ratio) which energizes the vortex above the main wing and delays bursting.

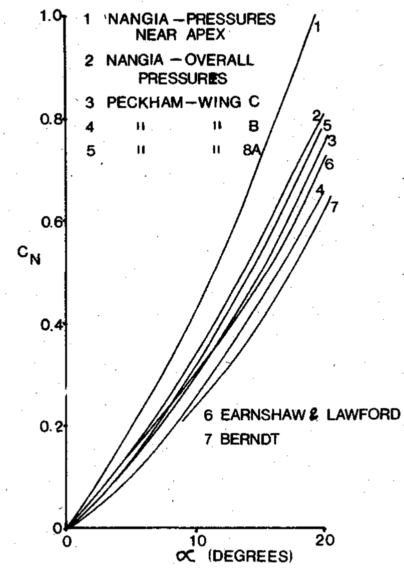

Fig. 6 Measured normal force coefficients.

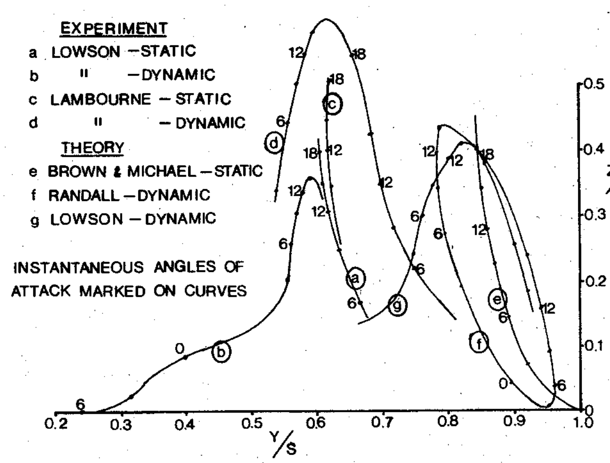

Fig. 7 Vortex loci—pitching oscillations.

VI. Theoretical Methods for Unsteady Flow

A delta wing with detached flow is a nonlinear system, and therefore it is not possible to build up a general time-dependent response from basic simple harmonic or indicial motions. Despite this, some attempts have been made to extend and apply the steady-state theoretical methods to specific unsteady motions.

A. Linearized Numerical Solutions

The extension of linearized lifting surface models to unsteady conditions has been discussed by Garner[17] and Watkins.[43] For oscillatory flow, both the local downwash and loading can be expressed as functions of x and y multiplied by $e^{i\omega t}$. This results in an integral equation relating the loading to the downwash, and involves a complicated kernel function that depends on position and frequency.

Exact solutions have been obtained for wings of circular and very slender (aspect ratio ≈ 0.001) planforms, but for more general planforms the equation has to be solved by numerical techniques. For low frequencies one of the most widely used techniques is that of Garner.[17]

B. Detached Flow Theories

A representative theoretical model for the flow pattern, incorporating leading-edge separation for slender wings undergoing unsteady motions is obviously not feasible in our present state of knowledge. Randall[44] obtained a solution for a slender, flat, delta wing undergoing sinusoidal oscillations in a steady stream, based on the model of Brown and Michael. By assuming the amplitude of oscillation to be small compared to the mean angle of attack, the resulting motion is reduced to a linear perturbation of a steady motion.

Randall presented results for the path of the vortex above a delta wing, of semiapex angle 10° oscillating in pitch about an axis 0.625 c from the apex, with angle of attack varying from 0° to 17.8° (Fig. 7).

In a latter paper Randall[45] extended his solution, using Smith's method to allow for wings with curved leading edges and longitudinal camber.

Working along similar lines, Lowson[10] obtained solutions for a flat delta wing undergoing both pitching and heaving motions. The analysis is more detailed then Randall's, and the assumptions of linear perturbations are not applied. Lowson determines the solution in terms of the passage of an effective camberline through a section of air. Instantaneous forces on the wing are obtained by integration of the forces per unit chord at each section of the fluid at the instant considered.

In his results emphasis again was placed on the motion of the vortex cores, but some results for the lift and moment were presented for both pitching and heaving motions. Results for the motion of the vortex core for an identical pitching motion to that considered by Randall were presented (Fig. 7). An interesting point in these results is that, if the angle of attack becomes zero at some point on the cycle, the vortex moves rapidly inboard and collapses; a new vortex then is generated near the leading edge. Unfortunately no experimental results are available for comparison, and so the accuracy of the force results cannot be assessed, but they are unlikely to be good because of the steady-state inaccuracies in the model.

For the indicial problem, Dore[46] studied the cases of a slender, flat, delta wing entering a sharp-edged gust and undergoing a sudden plunging motion. A similar approach to that of Lowson was used, reducing the problem to the passage of a section of air over a camber surface. The solution then is trivial for the sharp-edged gust. Each transverse station on the wing is unaffected until it reaches the gust front, when the loading associated with the new effective angle of attack is taken up immediately.

The sudden plunging motion is more complex; vorticity is generated along the entire leading edge at time $t=0$ and convected downstream at the freestream velocity. If the initial angle of attack is assumed to be zero, the problem reduces to that of obtaining the solution to the passage of a series of trapesium wings which was solved by Dore. Starting at the leading edge at time $t=0$ the vortex moves inboard and upwards, reaching its final steady position after a time $(Ut/x)=1$.

VII. Experimental Results in Unsteady Flow

A. Results for Oscillatory Motions

1) Force Measurements

One set of equipment used in the Royal Aircraft Establishment wind tunnels to obtain aerodynamic derivatives has been described by Thompson and Fail.[47] A pivoted model was excited continuously in pitch (or yaw) by an electromagnetic vibration generator. A spring unit, balanced the inertia loads and relieved the excitation forces which were measured together with the model displacement. The direct and cross derivatives were found from these measurements. Results obtained by this method on an ogee wing of aspect ratio 0.925 showed that the model was unstable in pitch, and the instability increased with angle.

Woodgate[48,49] has measured the pitching moment derivatives on slender delta wings in a low-speed wind tunnel, applying the method of inexorably forcing the model in pitch. Measurements were made of the in and out of phase force components, with the driving system wind on and off. The derivatives were determined from these. Results for delta wings with both sharp and round leading edges covered the angle of attack range between 0° and 15° and the frequency parameter $(\omega c/2U)$ range of 0.2 to 1.0. A fixed amplitude of oscillation of 1° was used. Results for a sharp-edged delta of aspect ratio 0.654 showed the stiffness and damping to be positive, with little variation with frequency.

2) Pressure Measurements

Laidlaw and Halfman[50] measured the unsteady pressures on a series of rigid wings of rectangular and delta planform, oscillating purely in pitch or heave. Mean angles of attack for both pitching and heaving modes was zero. Values of frequency parameter were in the range 0.3 to 0.6 at a Reynolds number of 2.1×10^6. Agreement was fair for the chordwise loading, total lift, and pitching moment with slender wing theory for the delta wings of aspect ratio 2.51.

3) Measurement of Vortex Positions

Most of the measurements of vortex positions so far have been obtained in water tunnels. Lowson[10] used fluorescein dye and a flash camera to determine the motion of the vortex cores relative to a delta wing oscillating in pitch and heave, whereas Lambourne obtained movie film records of the tracks of suspended particles.

Both workers obtained measurements for a delta wing, semiapex angle 10°, performing pitching oscillations identical to those theoretically analyzed by Randall and Lowson (Fig. 7). Discrepancies between the experimental results are large; Lambourne's results lie much higher and have a greater spanwise variation. However, the discrepancies are compatible with steady measurements made by Lambourne[38] (Fig. 5).

Both of the experimental loci lie well inboard of the theoretical curves because of the inherent limitations of the model. Qualitatively, Lowson's theory gives an excellent prediction of the nature of the vortex motion; the collapse predicted by the theory, as zero α is approached, is confirmed by his experimental results.

B. Results for Indicial Motions

Experimental measurements of the motion of the vortex cores above a delta wing of semiapex angle 10° undergoing a sudden plunging motion in a water tunnel have been made by Lambourne, Bryer, and Maybrey.[9] A movie camera technique similar to that for the pitching oscillations was used. Soon after the start of a plunge from zero α, a weak vortex is formed close to the leading edge at all chordwise stations. As the vortex strength grows, the core moves inboard and upwards. Very little movement occurs after a time $Ut/x=1$. Results for a wing plunging from an angle of attack of 0° to 11.3° showed poor quantitative agreement with Dore's theory, although the general trends in the motion are well-predicted.

VIII. Conclusions and Recommendations

The general nature of the steady flow past slender wings with sharp leading edges is well-understood, and the effects of aspect ratio, thickness, and leading-edge shape on the load distribution and overall forces are well-documented. Unfortunately, most of the experimental work to date has been done at Reynolds numbers considerably below those encountered in flight. Although changes in Reynolds number mainly seem to effect the secondary and tertiary separation and not the overall primary flow or loads, these effects do warrant further research.

Considerable efforts have been made to establish reasonable theoretical models, in addition to the large amount of experimental work that has been done. Unfortunately, none of the models so far available adequately predicts all aspects of the flow process.

In terms of its simplicity and accurate predictions of overall forces, Polhamus' leading-edge-suction analogy is excellent. However, like the lifting surface theories, it cannot accurately predict the spanwise loading. Of the more representative models, that of Mangler and Smith, gives the most useful approximations to the spanwise loading; however, because of its conical assumptions it does not allow for trailing-edge effects in subsonic flow. This failing is overcome by the somewhat simpler model of Nangia and Hancock, but much numerical work is required for still doubtful accuracy. However, this model is closer than most in its physical apporach, and is worthy of further development.

Most work has been done on low aspect ratio wings (less than 2), but significant improvements to landing performance and buffet boundary could be realized if methods of establishing stable tightly coiled leading-edge vortices on wings with lower sweep angles could be found. Work that has been conducted to date in this field should be continued and expanded.

With regard to the effects of unsteady flow on wings with leading edge vortices, few detailed systematic studies have been made either theoretically or experimentally. The problems involved are numerous but with faster computers and more sophisticated experimental techniques these could and should be overcome.

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